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JET FLOW AROUND A SPHERE

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ABSTRACT. The reasons for the stable behavior of a sphere in a thin jet of gas or liquid directed vertically upward are examined on the basis of previous studies and the author's own experiments. By conceiving of the jet as diverging ahead of the sphere and converging behind it, the problem becomes one of determining the characteristics of boundary layer flow which keep the sphere within the jet.

A sphere located in a thin jet of gas or liquid directed vertically upward is held stably in the latter. At a certain ratio between the size of the jet and the size of the sphere, however, the stability is destroyed and the sphere is ejected from the jet. /76*

It is interesting to explain the reason for the stable behavior of the sphere in the jet to determine the force that keeps it in a state of stable equilibrium, and to determine the value of the ratio of the half width of the jet to the radius of the sphere at which the latter is ejected from the jet.

The question of the stability of a sphere in a thin vertical jet is discussed in [1] in conjunction with flow around a circle, where the hypothesis is expressed that the point of divergence and the point of convergence of the jet lie on the same diameter. Without a hypothesis of this kind, the problem has no single solution within the limits of the scheme of an ideal liquid.

Jet flow around a body of blunt shape, and particularly around a sphere, was discussed in [2]. In this paper, the flow around bodies whose dimensions exceeded those of the nozzle was investigated at distances from the initial cross section of the jet up to 30-40 diameters of the body. It was confirmed experimentally that the flow around the sphere is continuous at distances up to 8-10 diameters. It is suggested that the continuous flow in this area is caused by splitting of the jet into two narrow semifinite jets. As a result

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*Numbers in the margin indicate pagination in the foreign text.

of the development of pressure differentials (an atmospheric one at the outer limit, and a negative one at the surface of the body) the jets are pressed against the surface of the body and flow continuously around it [3].

In the present paper, we shall be considering the nature of flow around a sphere suspended in a vertical, axially symmetric jet, with central flow around the sphere. The diameters of the spheres used were 74 and 37 mm. The nozzle dimensions varied from 6 to 74 mm.

The field of velocities behind the sphere, generally speaking, is determined by four parameters: the velocity of the incident flow, the nozzle radius, the size of the body and the distance from the nozzle opening to the cross section where the foremost point of the sphere is located. Due to the affinity of the velocity profiles in the different cross sections of the free jet [4], we can use the value Y as the characteristic width of the jet, which represents the distance from the axis to the point at which the velocity is equal to half the axial velocity in a given cross section.

The results of the experiments show that the nature of the flow in the wake behind a body in the immediate vicinity of the sphere (0.054 diameters) depends on the ratio Y/R. The value of Y is taken in the cross section where the foremost point of the sphere is located, and R is the radius of the sphere.

If $Y/R < 1$, the flow is continuous and the velocity profile has the form shown in Figure 1 (Curve 1) for $Y/R = 0.485$. With increasing distance from the body, the profile levels off and at a distance of 0.5 diameters is similar to the velocity profile of a free jet. If $Y/R \geqslant 1$, the profile has the form shown in Figure 1a (Curve 2) for $Y/R = 1.19$. A zone of reverse currents is visible. As Y/R increases, the dimensions of the circulation zone increase, approaching the dimensions of the zone in the case of flow of a uniform jet around a sphere.

The velocity profile behind the sphere is generalized for different nozzles, if Y/R = const (Figure 1a, Curves 1 and 2).

A change of the initial velocity of the incident flow within limits of 25 to 75 m/sec at constant Y/R does not change the picture of flow around the sphere.

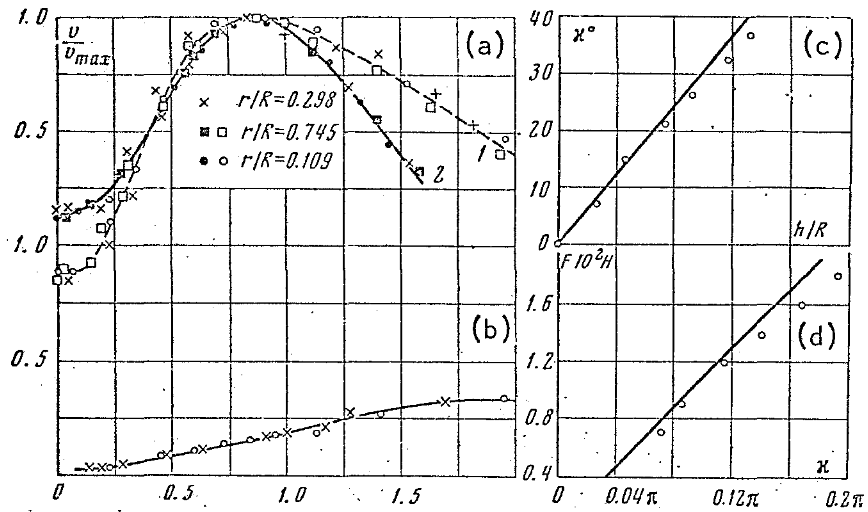

Figure 1.

We examined the kinematics of motion and found that the flow picture depends considerably on one parameter. It is natural to assume that the dynamics of motion also depend on this parameter. Experiments with a suspended sphere make it possible to determine easily the resistive force of the sphere, which is equal to its weight. Using the method of aerodynamic suspension of the sphere without weights and assuming the rate of flow to be equal to the average velocity in the cross section where the sphere is suspended, we obtain the coefficient of resistance of the sphere:

$$\zeta = 8mg / \pi d^2 \rho v^2 .$$

Here m is the mass of the spheres, d is its diameter, v is the average velocity, ρ is the gas density, g is the acceleration due to gravity, depending on the dimensionless parameter Y/R.

At $Y/R < 1$ the coefficient of resistance is low. With an increase of this parameter, ζ increases monotonically and tends toward the value of ζ in the case of flow around a sphere by a uniform jet (Figure 1b). The sphere then becomes unstable in the jet and is ejected from it. It was not possible to determine exactly the value of Y/R at which such a phenomenon is observed. The approximate value of the ratio is 2·5 to 3.0. /77

A change in the Reynolds number N_{Re} within the limits $6.7 \cdot 10^4$ to $2.0 \cdot 10^5$ has a weak influence on the coefficient of resistance, which remains practically constant in the given range of Reynolds numbers N_{Re} if Y/R = const.

Now let the axis of a thin jet be displaced relative to the center of the sphere. Then the picture of the flow of an ideal liquid in the vicinity of the point of convergence of the jets must be symmetrical with the picture of the flow in the zone of divergence of the jet. The jet is deflected by the sphere at a certain angle, and a stabilizing force is developed, directed toward the axis of the jet and returning the sphere to a state of stable equilibrium. We have determined experimentally the angles κ of deviation of the axis of the jet from the vertical behind the sphere in the case of non-centralized flow around the sphere (Figure 1c) as well as the force F drawing the sphere inward (Figure 1d).

If the axis of the jet passes through the center of the circle about which the flow passes, then the point of convergence is located one diameter from the point of divergence of the jet. It is natural to assume that, as in the case of central flow about a circle, we can consider in the first approximation that the critical points are located on the same diameter if the axis of the jet is displaced relative to the center of the sphere [1]. The hypothesis of [1] is supported by the experiments (Figure 2).

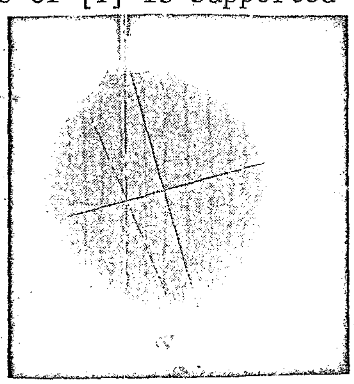
Figure 2.

The experimental facts that have been mentioned in conjunction with the hypothesis [1] allow us to construct a model of flow around a sphere, allowing an approximate calculation. Application of the theorem of impulses provides a possibility of determining the force that returns the sphere to a state of stable equilibrium. It is necessary to know the angle of deviation of the axis of the jet from the vertical behind the sphere in the case of non-central flow, as well as the radius of the divergent jet.

The hypothesis in [1] makes it possible to determine the angle of deviation of the axis of the jet from the vertical.

Let us consider the case of the flow of a thin jet around a circle. To determine the angle of deviation, it is necessary to estimate the distance of the point of divergence from the axis of the jet. With this goal in mind, let us consider the impact of the jet on a plate [5]. With a small width of the jet relative to the diameter of the sphere in the vicinity of the diverging point, it is possible to consider the adjacent arc of the circumference as a straight line tangent to the surface of the sphere at the point of its intersection with the axis of the jet.

In such a view, the distance of the point of divergence from the axis can be represented in the form

$$l = 2.3\ \beta d_0, \qquad \beta = {}^1/_2\ \pi - \alpha \tag{1}$$

where d_0 is the diameter of the jet and α is the slope angle of the axis of the jet relative to the surface. The expression is valid for small angles β. In addition, from the reversibility of motion it follows that the flow in the vicinity of the point of convergence of the jets behind the body must be symmetric with the flow in the zone of divergence of the jet, since the flow will be symmetric relative to some straight line which passes through the center of the circle and is perpendicular to a diameter drawn through the critical points. /78

Let us take the hypothesis of [1] as a basis and use (1); we can then calculate the angle of deviation of the axis of the jet from the vertical:

$$\varkappa = 2\,[\arcsin\,(h / R) - \arctan\,(l / R)] \tag{2}$$

Here h is the displacement of the axis of the jet relative to the center of the circle, R is the radius of the circle around which the flow is passing, l is the distance from the axis of the jet to the point of divergence along the arc of the surface around which the flow occurs, and κ is the angle of deviation of the axis of the jet from the vertical.

For a correct determination of the force acting on the sphere, the geometric picture is not completely acceptable, since the convergent jet is wider than the initial one owing to the effects of viscosity. For quantitative estimates it is necessary to take the narrowing of the jet into account. Let us express the radius of the convergent jet through the initial data: r_0, v_0, and R (the initial radius of the jet, the velocity of the incident flow, the coefficient of kinematic viscosity, and the radius of the sphere). We run up against the boundary layer theory. In the flow of a thin jet around a circle, the free surfaces of the divergent jets may be assumed to be the circumferences of a radius close to the radius of the surface around which the flow takes place. In this case, the velocity at the outer limit of the boundary layer will be an unknown quantity; therefore, we must add to the equations of the boundary layer an equation for constancy of divergence. The system of equations in coordinates s, n (s is the longitudinal coordinate along the contour, n is the transverse coordinate, calculated along the normal to the profile) assumes the form

$$v_s \frac{\partial v_s}{\partial s} + v_n \frac{\partial v_s}{\partial n} = \nu \frac{\partial^2 v_s}{\partial n^2}$$

$$\frac{\partial v_s}{\partial s} + \frac{\partial v_n}{\partial n} = 0 \tag{3}$$

$$G = 2\pi R \sin \frac{s}{R} \int_0^\delta v_s \, dn \tag{4}$$

Using the identity

$$v_n \frac{\partial v_s}{\partial n} = \frac{\partial (v_n v_s)}{\partial n} - v_s \frac{\partial v_n}{\partial n},$$

the first equation (3) is rewritten as follows:

$$\frac{\partial v_s^2}{\partial s} + \frac{\partial (v_n v_s)}{\partial n} = \nu \frac{\partial^2 v_s}{\partial n^2} \tag{5}$$

We now introduce the designation $n/\delta = \eta$, where δ is the thickness of the small jet flowing around the sphere. Integrating equation (5),

$$\frac{d}{ds} \int_0^\delta v_s^2 \, dn = \nu \left. \frac{\partial v_s}{\partial n} \right|_0^\delta \tag{6}$$

We express the velocity in the form /79

$$v_s = a\eta + b\eta^2 + c\eta^3$$

$$\partial^2 v_s / \partial \eta^2 = 0 \quad \text{when} \quad \eta = 0, \qquad b = 0$$

From the boundary conditions, we find the coefficient c:

$$c = -\,{}^1/_3\, a, \; [\partial v_s / \partial \eta]_{\eta=1} = 0, \qquad v_s = a\,(\eta - {}^1/_3\, \eta^3)$$

Considering that at point s_0 the velocity is equal to the initial $v(s_0) = v_0$, $s_0 \approx r_0$, we write $a(s_0) = 3/2\ v_0$ (Figure 3).

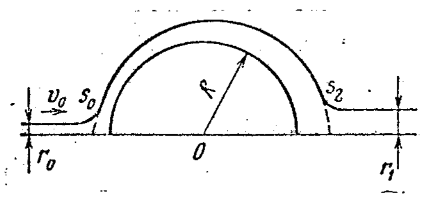

Figure 3.

The problem becomes one of finding a and δ; the latter can be determined by two equations, obtained from (6) and (4) as substituted for the expressions for v_s:

$$\frac{d}{ds} a^2\delta \int_0^1 \left(\eta - \frac{1}{3}\eta^3\right)^2 d\eta = -\nu \frac{a}{\delta} \tag{7}$$

$$2\pi R \sin \frac{s}{R} \delta a \int_0^1 \left(\eta - \frac{1}{3}\eta^3\right) d\eta = G \tag{8}$$

or

$$\frac{d}{dx}(a^2\delta) = -\frac{\lambda a}{\delta}, \qquad a\delta = \frac{k}{\sin x}$$

$$\frac{s}{R} = x, \qquad \frac{315\nu R}{68} = \lambda, \qquad \frac{6G}{5\pi R} = k$$

Omitting δ, we will have

$$\frac{d}{dx}\frac{ka}{\sin x} = -\lambda \frac{a^2 \sin x}{k},$$

After several transformations, we obtain the differential equation

$$\frac{dy}{dx} = -my^2 \sin^3 x \qquad \left(y = \frac{a}{\sin x},\ m = \frac{\lambda}{k^2}\right)$$

It has the solution

$$1/y = m\left(\tfrac{1}{3}\cos^3 x - \cos x\right) + C$$

Satisfying the original condition $a(x_0) = 3/2\ v_0$, $x_0 = s_0/R$, we find a as follows:

$$a = \frac{\sin x}{m\left(\tfrac{1}{3}\cos^3 x - \cos x\right) + \tfrac{2}{3}\left((s_0/v_0R) + m\right)}$$

Consequently,

$$a_1 = \frac{\sin x_1}{m\left(\tfrac{1}{3}\cos^3 x_1 - \cos x_1\right) + \tfrac{2}{3}\left((s_0/v_0R) + m\right)}$$

Considering that $x_1 = \pi - x_2$ for small x_2, where $x_2 \approx r_1/R \approx s_2/R$, r_1 being the final radius of the jet, we find a_1 in the form

$$a_1 = \frac{x_2}{\tfrac{2}{3}(2m + s_0/v_0R)}$$

The deviation for the initial cross section of the jet is expressed as $G = \pi r_0^2 v_0$, for the final cross section $G = \pi R^2 x_2^2\ 2/3\ a_1$. Proceeding from the condition of constancy of deviation, we equate these two expressions to one another and find the value of the final radius of the jet through the initial parameters /80

$$r_1 = \left(r_0^3 + \frac{6.45\nu R^4}{r_0^2 v_0}\right)^{1/3} \tag{9}$$

Using the theorem of the preservation of momentum and using expressions (2) and (9), we calculate the reaction of the jet to non-centralized flow around a sphere:

$$\int \rho v v_n d\sigma = \mathrm{F}$$

The force that returns the sphere to a state of equilibrium is directed along the x-axis and is expressed as follows:

$$F_x = \pi\rho r_0^2 v_0^2 (r_0 / r_1)^2 \sin \varkappa$$

The force of resistance of the sphere in the jet is

$$F_y = \pi\rho r_0^2 v_0^2 [1 - (r_0 / r_1)^2]$$

The coefficient of resistance calculated by formula

$$\zeta = 8F_y / \rho v_0^2 \pi d^2$$

for a thin jet coincides with the experimental data. The calculated data for the angle of deviation of the axis of the jet from the vertical and the stabilizing force at various angles of deviation is completely and reliably in agreement with the experimental findings (Figure 1c and 1d).

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